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SUBJECT/GRADE LEVEL

Subject Area: Algebra II, Physics (Mathematical Modeling Emphasis)

Grade Level: High School

Materials

Classroom Supplies:

- Video/animation of horizontal projectile (e.g. Veritasium)
- Meterstick or measuring tape
- Stopwatch or timer (optional)
- Student experiment worksheet (data table)
- Graph paper or digital plotting tools
- Chart paper or large whiteboards for group sharing
- Projector or display screen for visuals
- Markers, pens, colored pencils

Lab Supplies (per group):

- 3 small balls (ping pong, metal, plastic)
- A table or desk for launching (75 – 100 cm high)
- Carbon paper or masking tape to mark landing location
- Safety goggles (optional but encouraged)

Digital Tools (optional but helpful):

- Desmos or GeoGebra (for graphing and regression)
- Slow-motion video app (to visualize projectile motion)
- Canva/Google Slides for student presentations

STANDARDS

Next Generation Science Standards

- HS-PS2-1: FORCES AND INTERACTIONS
  Analyze data to support the claim that Newton's second law of motion describes the

mathematical relationship among the net force on a macroscopic object, its mass, and its acceleration.

- HS-PS2-2: MOTION AND STABILITY
  Use mathematical representations to support the analysis of an object's motion under the influence of gravitational force.

Mathematics Standards | Common Core State Standards

- CCSS.MATH.CONTENT.HSF.IF.B.4
  Interpret key features of graphs and tables in terms of the quantities they represent, and sketch graphs showing key features given a verbal description of the relationship.

- CCSS.MATH.CONTENT.HSF.BF.A.1
  Build a function that models a relationship between two quantities.

- CCSS.MATH.CONTENT.HSN.Q.A.1
  Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently.

- CCSS.MATH.CONTENT.HSN.VM.B.4
  Add and subtract vectors; represent vector quantities using appropriate notation and apply vector addition in context.

English Language Arts Standards | Common Core State

- CCSS.ELA-LITERACY.SL.11-12.1.A
  Come to discussions prepared, having read and researched material under study; explicitly draw on that preparation by referring to evidence from texts and other research on the topic or issue to stimulate a thoughtful, well-reasoned exchange of ideas.

- CCSS.ELA-LITERACY.RST.11-12.7
  Integrate and evaluate multiple sources of information presented in diverse formats and media in order to address a question or solve a problem.

LESSON OBJECTIVE(S)

By the end of this unit, students will be able to:

- Model the horizontal displacement of a projectile using dimensional reasoning

- Decompose vector motion into independent horizontal and vertical components

- Analyze real and simulated data to discover mathematical patterns
- Construct and interpret functions relating velocity, time, and distance
- Apply unit consistency checks to validate physical models
- Derive proportional relationships between physical quantities using the Rayleigh method of dimensional analysis
- Collaborate to design and carry out a hands on experiment to collect and evaluate data
- Compare their own mathematical models with those found in current scientific literature
- Communicate findings using appropriate mathematical and scientific language

Unlike traditional lessons on projectile motion, this unit challenges students to rediscover the relationships behind motion through dimensional analysis, data modeling, and vector reasoning not through pre-given formulas. The approach is inspired by original research, including a peer-reviewed article by the instructor's mentor, and encourages students to think like applied scientists rather than passive learners.

CAREER CONNECTION(S)

Mentor Background: Dr. Mark Ginsberg Dr. Ginsberg is a Principal Investigator in the Materials and Structures Branch, Facilities Division, at the U.S. Army Engineer Research and Development Center (ERDC), Construction Engineering Research Laboratory (CERL) in Champaign, Illinois. He applies control theory and mathematical modeling to analyze complex physical systems. His interdisciplinary work, bridging physics, mathematics, and engineering, inspires this lesson's focus on dimensional analysis, vector decomposition, and real-world problem-solving through modeling and simulation.

STEM Careers:

- Control Systems Engineer
- Data Scientist
- Structural Analyst

ENGAGEMENT

Brief Description: Students are introduced to a surprising physics paradox: two bullets one dropped vertically and one launched horizontally hit the ground at the same time. A video demonstration sparks curiosity and cognitive dissonance, prompting students to question assumptions about motion. This serves as the foundation for a deeper modeling investigation.

Procedures: Present the following scenario to students:

Imagine two bullets released at the same moment from the same height—one is dropped straight down from the top of a cliff, and the other is fired horizontally from that same height. Which bullet hits the ground first? This classic physics question challenges intuition and sparks curiosity.

Prediction and Group Discussion (Before Video):

- Have students work in small groups to discuss and record their predictions.
- Encourage them to explain their reasoning: Which bullet do they think hits first? Why?
- Ask groups to consider factors that might influence the bullets' motion and impact time.

Video Demonstration:

- Show a short slow-motion video or animation demonstrating the experiment, such as Veritasium's "The Bullet Drop."
- Students observe the actual outcome, which reveals that both bullets hit the ground simultaneously.

Re-examination and Group Reflection (After Video):

Students return to their groups to compare the video results to their predictions.

- Facilitate discussion around any surprises or questions raised.

Whole-Class Debrief:

- Guide a class discussion to address misconceptions and deepen understanding.
- Emphasize the concept that horizontal and vertical motions are independent.

- Help students see that gravity affects both bullets equally in the vertical direction and that horizontal velocity does not influence the time to fall.

Exploratory Questions for Discussion:

- Which bullet hits the ground first? Why?
- What factors might influence how far the fired bullet travels?
- Does horizontal speed affect falling time? Why or why not?
- What is the role of gravity in this situation?
- Can we use only mathematics to predict this outcome?

Introduce the Guiding Question:
Conclude the engagement by introducing the unit's guiding question to launch the deeper investigation:
"How can we use dimensional reasoning and mathematical modeling to understand a physical system like this without simply memorizing formulas?"

- Student Prediction Sheet

EXPLORATION

Brief Description: Students investigate the motion of a horizontally launched projectile through guided experimentation and mathematical modeling. Using given parameters (initial velocity and height), they collect data and apply vector decomposition and dimensional analysis to derive a model for range. The Rayleigh method is introduced as a tool to derive proportional relationships between physical quantities without relying on pre-given formulas. The activity emphasizes real world reasoning over formula memorization.

Procedures:

Class Hour 1 – Launch & Set-Up

Objective: Generate curiosity and prepare for data collection through inquiry.

Student Activities:

- Watch Veritasium's *"The Bullet Drop"* or Shot VS Drop and make predictions
- Discuss assumptions in small groups and revise their ideas
- Reflect in writing: *"What variables affect how far something flies? What causes something to fall?"*
- Identify what should be measured in the upcoming experiment
- Assist in preparing experiment stations (measuring height, clearing launch path)

Teacher Tasks:

- Facilitate discussion and clarify the concept of two-dimensional motion
- Introduce key guiding question:
  "Can we explain this motion mathematically without memorizing formulas?"
- Ensure each group has set up materials ready

Associated Resources:

- Bullet Drop Video Link & Shot VS Drop
- Group Reflection Prompt Sheet

Class Hour 2 – Experimental Data Collection

Objective: Observe horizontal projectile motion and collect range and time-related data.

Student Activities:

- With guidance, they observe that the range R relates to horizontal speed V_0 and fall time t.

Teacher Tasks:

- Circulate for support and safety supervision

- Help students accurately measure distances
- Prompt them with guiding questions:

  "Why do you think the fall time is the same for each?"

  "How can you tell which trial was fastest?"

Associated Resources:

- Explore Data Table , Example Data Table – Ball Launch Trials
- Fired vs. Dropped Objects: Student Procedure Guide
- Group Trial Log Sheet
- Materials: balls, ruler/tape, tubing, launch area (Materials List)

Class Hour 3 – Graphing & Pattern Analysis

Objective: Analyze results and model the relationship between velocity, height, and range.

Student Activities:

- Plot R ve V_0 using their data
- Sketch model relationship
- Begin generalizing pattern:

  $$R = v_0 \cdot \sqrt{\frac{h}{g}}$$
- Review unit types (m, s, m/s) and perform dimensional checks

Teacher Tasks:

- Support graphing skills (or use Desmos/Excel)
- Guide conversations around unit analysis and vector notation
- Introduce conceptual vector form:

$$\vec{v_0} = \langle v_0, 0 \rangle, \quad \vec{a} = \langle 0, -g \rangle$$

Associated Resources:

- Graphing Guide Sheet
- Modeling with Vectors Worksheet

Class Hour 4 – Dimensional Reasoning & Reflection

Objective: Validate the model with dimensional analysis and prepare for extension.

Student Activities:

- Confirm model via unit analysis
- Compare results with other groups
- Write reflection: *"What determines the projectile's range?"*
- Use exit ticket to assess concept understanding

Teacher Tasks:

- Guide final model justification using units
- Facilitate discussion around why motion is separable
- Assign pre-reading of professor's article excerpt

Associated Resources:

- Dimensional Analysis Sheet
- Student Worksheet – Horizontally Ejected Bullet Investigation
- Exit Ticket: "What did you learn about horizontal vs. vertical motion?"

EXPLANATION

Brief Description: Students will construct a mathematical model based on their own experimental data, demonstrate an understanding of vector-based decomposition of motion, and verify the model using dimensional analysis. This phase transitions student thinking from pattern recognition to formal mathematical representation.

Procedures:

1. Model Construction (Whole Class/Small Group):

Students revisit their data from the EXPLORE phase.

- With guidance, they observe that the range R relates to horizontal speed V_0 and fall time t.
- They recall or are guided to derive:

$$R = v_0 . t \quad \text{and } t = \sqrt{\frac{2h}{g}}$$

- Substituting, they arrive at:

$$R = v_0 . \sqrt{\frac{2h}{g}}$$

2. Vector-Based Reasoning:

- The teacher introduces or reviews motion vectors:

$$\overrightarrow{v_0} = \langle v_0, 0 \rangle, \quad \vec{a} = \langle 0, -g \rangle$$

- Students graphically represent vectors and identify how horizontal motion is unaffected by vertical acceleration.

3. Dimensional Analysis:

- Students check unit consistency:
  - RHS: $[v_0] = \mathrm{m/s}, \quad [\sqrt{h/g}] = \sqrt{\mathrm{m}/(\mathrm{m/s^2})} = \mathrm{s}$
  - So: $R = \mathrm{m/s} \cdot \mathrm{s} = \mathrm{m}$
- This reinforces that the model is valid in terms of dimensions.

4. Class Discussion & Clarification:

- The teacher prompts:

  "What does each term represent?"
  "Why is motion in one direction independent of the other?"
  "How do we know this model is reliable?"

Teacher's Role:

- Facilitate connection from observation to modeling
- Provide scaffolds for vector and dimensional reasoning
- Use formative questioning to ensure conceptual clarity

Student Outcomes:

- Accurately express motion using vectors
- Justify a physical model with both data and units

- Explain the relationship between range, velocity, and time in their own words

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Associated Resources:

- Modeling with Vectors Worksheet
- Graphs from
- Article excerpt on dimensional modeling
- Teacher Formula Reference Card

ELABORATE

Brief Description: In this phase, students go beyond applying a formula they revisit the reasoning behind the model using dimensional analysis. The goal is for students to re-derive the relationship independently, focusing on units and logical structure rather than memorization.

Procedures:

Activities:

- In small groups, students are given the challenge:
  "How does range R depend on launch speed V_0 drop height h, and gravity g?"
- They are introduced (or reminded) of the Rayleigh method and asked to assume a general model:

$$R = C.v_0^a.h^b.g^c$$

Students write the dimensional representations of the variables:

$$[R] = L,\ \ [v_0] = \frac{L}{T},\ [h] = L,\ \ [g] = \frac{L}{T^2}$$

- Range is directly proportional to the initial velocity and also depends on the square root of the height divided by gravitational acceleration.
- Students compare the result to the earlier experiment-based formula and discuss consistency.
- Groups are encouraged to test variations in h, V_0, and even consider theoretical cases like other planets (with different g) to generalize the model further.

Guiding Questions:

- What mathematical form did you find for the relationship between R, V_0, h, and g?

- Why is gravity in the denominator? Why is height under a square root?
- How would this model behave if launched on the Moon?
- What important effects (like air resistance) are missing from our model?

Purpose:
To shift students from applying formulas to understanding and generating models based on unit structure, logical proportionality, and mathematical reasoning. This reinforces both physical intuition and algebraic fluency.

EVALUATION

Brief Description: Students act as mathematical modelers to apply their final equation for the range of a horizontally launched object. Using new conditions, they must test the accuracy and limitations of their model. Students will justify their reasoning, interpret data, and reflect on what the model captures and what it leaves out.

Procedures:

Math Class:

- Students are given 2 new scenarios with different heights and initial velocities.
- Using their derived model

$$R = v_0 \cdot \sqrt{\frac{2h}{g}}$$

they calculate the expected range in each case.

- The teacher provides simulated "actual" ranges (with slight variation to simulate air resistance or measurement error).
- Students compare their predicted values with actual data and respond to:
  - How close was your prediction?
  - What assumptions might have caused a difference?
  - Would your model still work on the Moon or Mars?

Scientific Reflection:

- In small groups, students complete a Model Evaluation Sheet, which includes:

1. Listing strengths of the model
2. Identifying limitations
3. Suggesting how the model might be improved

- Optionally, students may prepare a short presentation or poster that includes:
    1. A labeled diagram of vector components
    2. A step-by-step outline of how they derived the model
    3. One real-world example where this model could be used (e.g., sports, engineering)

Possible Adaptations:

- Students could work individually or in groups depending on time and scaffolding needs
- Instead of a poster, students could record a short video explaining their model and its usefulness in a real world context
- Teachers may allow students to select different planetary gravities (e.g., Mars, Moon) to explore how g changes their predictions

APPENDICES

Theoretical Framework and Derivations

REFERENCES

1. **Ginsberg, M. (2014).** *Bioprotection of Facilities: A Modeling-Based Approach.* DOI: 10.1080/14751798.2014.995335
2. **Langhaar, H. L. (1951).** *Dimensional Analysis and the Theory of Models.* Wiley. PDF:
3. **Szirtes, T. (2007).** *Applied Dimensional Analysis and Modeling* (2nd ed.). Elsevier. PDF:
4. **Spiegel, M. R.** *Schaum's Outline of Vector Analysis and an Introduction to Tensor Analysis.* McGraw-Hill Education. Book Info: https://www.mheducation.com
5. **Kittel, C., & Kroemer, H.** *Thermal Physics* (2nd ed.). W.H. Freeman. ISBN: 9780716710882
6. **Lathi, B. P. (2005).** *Linear Systems and Signals* (2nd ed.). Oxford University Press. ISBN: 9780195158335

7. **Lathi, B. P., & Green, R. (2017).** *Linear Systems and Signals* (3rd ed.).
   Oxford University Press.
   ISBN: 9780190200176
8. **How to Become Quick at Figures** (1885).
   The Woodbury Company, Boston.
   Full Text: https://archive.org/details/howtobecomequick00wood
9. **Lumen Learning.** (n.d.). *Projectile Motion*. In College Physics. Retrieved from
   https://courses.lumenlearning.com/suny-physics/chapter/3-4-projectile-motion/